Since the tangent modulus stiffness a is positive, it is evident that, as n increases, the value of $\bar{x}^{(n)}$ approaches the exact value given by Eq. (5).

Reference

[1] Marcal, P. V., "A Comparative Study of Numerical Methods of Elastic-Plastic Analysis," *AIAA Journal*, Vol. 6, No. 1, Jan. 1968, pp. 157–158.

Reply by Author to M. A. Salmon

P. V. Marcal*
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THE lack of convergence of the constant strain approach for an elastic perfectly plastic material was argued for a two-dimensional constant stress element.[1] It is not possible to use Salmon's example of a one-dimensional constant stress element to test the above claim, since the material exhibits different behavior in the two different cases. Because of the normal flow rule of plasticity, the yielded material in two dimensions still possesses a certain amount of resistance to straining. This is reflected in the stress-strain relation $[P^-]$. This is not true in the one-dimensional case where $[P^-]$ is equal to zero.

Salmon's criticism does, in fact, raise an important point. Since most of the analysis was developed in general matrix form, it would be expected that, for an elastic perfectly plastic material, the lack of convergence should apply equally to all types of constant stress elements. However, this line of reasoning neglects the fact that $[P^-]$ does not exist for a one-dimensional truss element. Because $[P^-]$ does not exist, Eq. (13) of Ref. 1 is invalid, and the convergence study that is based on this equation can no longer be expected to hold.

Reference

[1] Marcal, P. V., "A Comparative Study of Numerical Methods of Elastic-Plastic Analysis," *AIAA Journal*, Vol. 6, No. 1, Jan. 1968, pp. 157–158.

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Comment on "A Formula for Updating the Determinant of the Covariance Matrix"

James E. Potter* and Donald C. Fraser†
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A FORMULA was presented in Ref. 1 for updating the determinant of the covariance matrix of state estimation errors when measurement statistics are incorporated. This result may be obtained in a much simpler way if a more general identity is first proved. This general identity is

$$|A||D + CA^{-1}B| = |D||A + BD^{-1}C| \tag{1}$$

where the individual matrices have the following form:

$A = n \times n$ (nonsingular) $\quad C = m \times n$

$B = n \times m \quad D = m \times m$ (nonsingular)

This identity is obtained by manipulating partitioned matrices as follows:

$$\begin{bmatrix} A & -B \\ C & D \end{bmatrix} \begin{bmatrix} I & A^{-1}B \\ O & I \end{bmatrix} = \begin{bmatrix} A & O \\ C & (D + CA^{-1}B) \end{bmatrix} \tag{2}$$

Since the second matrix on the left side of Eq. (2) has unity determinant, there results

$$\begin{vmatrix} A & -B \\ C & D \end{vmatrix} = |A| \; |D + CA^{-1}B| \tag{3}$$

. Similarly,

$$\begin{bmatrix} A & -B \\ C & D \end{bmatrix} \begin{bmatrix} I & O \\ -D^{-1}C & I \end{bmatrix} = \begin{bmatrix} A + BD^{-1}C & B \\ O & D \end{bmatrix} \tag{4}$$

$$\begin{vmatrix} A & -B \\ C & D \end{vmatrix} = |D| \; |A + BD^{-1}C| \tag{5}$$

Equating Eqs. (3) and (5) gives the identity of Eq. (1). Using the following change of vocabulary in Eq. (1):

$$A = P'^{-1} \qquad B = H \qquad C = H^T \qquad D = R \tag{6}$$

together with the optimum linear filter update equation

$$P^{-1} = P'^{-1} + HR^{-1}H^T \tag{7}$$

leads directly to the main result of Ref. 2,

$$|P|/|P'| = |R|/|R + H^TP'H| \tag{8}$$

By making the substitutions

$$D = I \qquad C = DC \tag{9}$$

Eq. (1) can be extended to the case where D is singular. The result is

$$|A| \; |I + DCA^{-1}B| = |A + BDC| \tag{10}$$

Reference

[1] Potter, J. E. and Fraser, D. C., "A Formula for Updating the Determinant of the Covariance Matrix," *AIAA Journal*, Vol. 5, No. 7, July 1967, pp. 1352–1354.

Received March 28, 1968.
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Comment on "Feasibility of a High-Performance Aerodynamic Impulse Facility"

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THE entropy correlation of the nonequilibrium chemical species concentrations of expanding air offered in Refs. 1 and 2 was based upon l values [i.e., $A/A^* = 1 + (x/l)^2$] of only 1 and 4.74 cm. Therefore, this correlation as reproduced in Ref. 3 is in error. The value of $l = 10$ cm shown in Ref. 3 appears to be a typographical error introduced when

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the original results[1,2] were transposed by the authors[3] and should be listed as $l = 1.0$ cm on their Fig. 3 in Ref. 3. Leonard and Rose[3] have therefore incorrectly used the entropy correlation to substantiate their otherwise valid conclusions regarding the need for longer nozzles at higher-enthalpy conditions at lower values of S/R (see Ref. 3, p. 450).

References 4 and 6 show that the entropy correlation[1,2] could be extended to account for geometric scaling. There is an accountability for the effect of enthalpy by relating it through flow velocity to chemical reaction time and thereby to a reaction distance or nozzle axial length x. It is therefore possible to extend the entropy correlation[1,2] and obtain a modified entropy correlation for the frozen oxygen atom composition and successfully apply it to data that vary by a factor of 20 in nozzle geometric scale. Other questions regarding very high-enthalpy performance and facility design criteria at low-entropy values have been treated in detail by Warren and Marston.[5]

The authors[3] also state that the work of Ref. 7 shows that the efficiency of arc heating helium increases with increasing gas density and the work of Ref. 8 shows it to decrease with increasing density. The work of Refs. 7 and 8 was performed by the same investigators. The earlier study[7] was conducted in a small 48 kjoule, 1.42-ft-long driver. Over the range of gas densities studied the efficiency did indeed increase. The work of Ref. 8 was performed in the 768 kjoule, 4.5-ft-long driver and over the range of gas densities studied the efficiency stayed fairly constant ($\sim$95%) and then began to drop off ($\sim$80%) at the higher gas density values. This is proper since it should not be expected that the efficiency will reach a constant value and remain constant with infinite increases in gas density, especially since the initiation of the arc discharge in the driver appears to have some density dependence.

References

[1] Harris, C. J., "Comment on 'Nonequilibrium Effects on High-Enthalpy Expansion of Air'," *AIAA Journal*, Vol. 4, No. 6, June 1966, p. 1148.

[2] Harris, C. J. and Warren, W. R., "Correlation of Nonequilibrium Chemical Properties of Expanding Air Flow," Rept. R64SD92, Dec. 1964, General Electric Co.

[3] Leonard, R. L. and Rose, P. H., "Feasibility of a High-Performance Aerodynamic Impulse Facility," *AIAA Journal*, Vol. 6, No. 3, March 1968, pp. 448–457.

[4] Harney, D. J., "Similarity of Nonequilibrium Expansions in Hypersonic Nozzles," FDM-TM-67-1, May 1967, Wright-Patterson Air Force Base, Ohio.

[5] Warren, W. R. and Marston, C. H., "A High Density, High Velocity Equilibrium Freestream Shock Tunnel Concept," Paper 68-17, Jan. 1968, AIAA; also Rept. R68SD1, Jan. 1968, General Electric Co.

[6] Ring, L. E. and Johnson, P. W., "Correlation and Prediction of Air Nonequilibrium in Nozzles," Paper 68–378, 1968, AIAA.

[7] Warren, W. R., Rogers, D. A., and Harris, C. J., "The Development of an Electrically Heated Shock Driver Test Facility," GE R62SD37, April 1962; also *Proceedings of the 2nd Symposium on Hypervelocity Techniques*, Denver, Colorado, March 1962.

[8] "Feasibility Study of a High Density Shock Tunnel Augmented by an MHD Accelerator," AEDC-TR-65-225, Oct. 1965, Arnold Engineering Development Center.